Econ4130
06 H

Exercises for week 48

There are no seminars week 48. Solutions to the exercises will be put on the net Wednesday 29 Nov.

1) **Rice ex. no 9.40 (9.30 in ed. 2).**
Note that the probabilities under H_0, p_1 and p_2 are meant to be known. This means that the number of free parameters under H_0 is 0, so the degrees of freedom for the χ^2-test is simply the number of categories minus 1.

2) **Rice ex. no 9.43 (9.33 in ed. 2).**

3) **Supplementary exercise 9** (as follows):

Suppose that $X_1, X_2, \ldots, X_m$ are independent and poisson distributed, where $X_i \sim \text{pois}(\mu_i)$ for $i = 1, 2, \ldots, m$. Let $N = X_1 + X_2 + \cdots + X_m$. Then, according to supplementary exercise 8c, $N \sim \text{pois}(\mu_1 + \mu_2 + \cdots + \mu_m)$.

Show that the joint conditional distribution of $X_1, X_2, \ldots, X_m$, given $N = n$, is multinomial $(n, p_1, \ldots p_m)$ where the cell probabilities are given by $p_j = \dfrac{\mu_j}{\mu_1 + \mu_2 + \cdots + \mu_m}$. [**Hint:** Note that

$$f(x_1, x_2, \ldots, x_m \mid n) = P(X_1 = x_1 \cap \cdots \cap X_m = x_m \mid N = n) = \frac{P(X_1 = x_1 \cap \cdots \cap X_m = x_m)}{P(N = n)}$$

where $x_1, x_2, \ldots, x_m$ satisfy $x_1 + x_2 + \cdots + x_m = n$. Explain the last equality for *f*.]

4) **Rice ex. no 9.36 (9.28 in ed. 2).**

Hint to Rice's hint: Rice suggests to model the frequencies in the table as multinomial. That presupposes that the sum, *N*, of the frequencies is given (i.e., non random). A rationale for this can be provided by problem 3) above. The term "monthly rate" is also slightly imprecise since the months vary somewhat in length. Also here we may use problem 3) to make the model and problem more precise:

Let X_j denote the frequency for month *j*, $j = 1, 2, \ldots, 12$. Assume that $X_1, X_2, \ldots, X_{12}$ are independent and $X_j \sim \text{pois}(\mu_j)$. We may now interpret the term "monthly rate" as the rate per 30 days, that we may call λ_j for month *j*, thus introducing an "average" month of 30 days. If d_j denotes the number of days in month *j*, then, for month *j*, μ_j / d_j gives the rate per day and hence $\lambda_j = 30 \cdot \mu_j / d_j$, the rate per 30 days. Or:

$$\mu_j = \frac{d_j}{30} \lambda_j, \quad j = 1, 2, \ldots, 12$$

Our interpretation of the problem is then to test the null-hypothesis

$$H_0: \quad \lambda_1 = \lambda_2 = \cdots = \lambda_{12} \quad \text{(call the common value } \lambda \text{)}$$

Now consider the conditional distribution of $(X_1, X_2, \ldots, X_{12})$ given that $N = n$, where $N = X_1 + X_2 + \cdots + X_{12}$, which, according to problem 3) is multinomial with parameters $(n; p_1, p_2, \ldots, p_{12})$. Show that the parameters, p_j, under H_0, are completely specified (known). Hence, the number of free (unknown) parameters of the multinomial model under H_0 (that we called *r* in the general theory) is 0 here. We have now enough information to perform a χ^2-test for H_0 based on the multinomial model. Perform the test and calculate the p-value. Interpret the results.